

the author's notation

$$\begin{aligned} \frac{k^2}{2} \frac{d\bar{\xi}}{d\tau_n} &= \ln c + \ln \bar{\xi} + 1 \\ \frac{k^2}{2} (\bar{\xi} - \bar{\xi}_0) &= (1 + \ln c) \tau_n + \int_0^{\tau_n} n \ln \left( \frac{2b}{k} \tau_n + \frac{\bar{Z}_0}{Z_n} \right) d\tau_n \\ &= (1 + \ln c) \tau_n \\ &+ \frac{k}{2b} \left[ \left( \frac{2b}{k} \tau_n + \frac{\bar{Z}_0}{Z_n} \right) \ln \left( \frac{2b}{k} \tau_n + \frac{\bar{Z}_0}{Z_n} \right) \right. \\ &\quad \left. - \left( \frac{2b}{k} \tau_n + \frac{\bar{Z}_0}{Z_n} \right) - \frac{\bar{Z}_0}{Z_n} \ln \frac{\bar{Z}_0}{Z_n} + \frac{\bar{Z}_0}{Z_n} \right] \end{aligned}$$

but

$$\bar{\xi}_0 = \frac{\bar{X}_0}{Z_n} = \frac{1}{k} \left( \frac{\bar{Z}_0}{Z_n} \right) \ln \frac{\bar{Z}_0}{z_0}$$

The corrected expression for  $c$  in terms of the author's notation should be

$$\begin{aligned} c = \exp \left\{ \frac{k^2 \bar{\xi}}{2\tau_n} - \frac{k}{2b\tau_n} \left[ \left( \frac{2b}{k} \tau_n + \frac{\bar{Z}_0}{Z_n} \right) \times \ln \left( \frac{2b}{k} \tau_n + \frac{\bar{Z}_0}{Z_n} \right) \right. \right. \\ \left. \left. - \left( \frac{\bar{Z}_0}{Z_n} \ln \frac{\bar{Z}_0}{Z_n} \right) - \frac{k}{2\tau_n} \left( \frac{\bar{Z}_0}{Z_n} \right) \left[ \ln \frac{\bar{Z}_0}{z_0} \right] \right\} \end{aligned}$$

The choice of scales selected by the author did not make clear the values of  $u_t$  or  $z_0$  actually utilized. Hence it is not possible to calculate whether the additional term fully compensates for the excursion in  $c$  found in Fig. 28. The correction would appear to be large and in the right direction.

In the case of stably stratified flow, an extension of the Lagrangian similarity for elevated sources such as is found in [4] should be used to develop the equation. It is found therein that  $d\bar{X}/dt - \bar{U}(\bar{Z})$  is always in an excess by an amount which has between  $u_t$  and  $2u_t$  for extreme values of stability. Because of its small magnitude, the error is significant only near the point of release and hence the assumption  $d\bar{X}/dt \approx \bar{U}(\bar{Z})$  is reasonable when considering diffusion at large distances.

It must, however, be emphasized that the expression is still approximate and to use this as a test is only appropriate for large diffusion times. Again the author's calculations fail due to the limited time considered. In the author's notation the dimensionless diffusion times required must be

$$t \gg \frac{h}{u_t} \text{ or } \tau_n = \frac{D_n t}{Z_n^2} \gg \frac{D_n h}{u_t Z_n^2} = 0.2 h \approx 0.02 \quad \text{say } \tau_n \approx 0.2;$$

whereas the maximum times displayed are  $\tau_n = 0.1$ . The situation is further frustrated by the fact that the numerical solution provided is developed for the bounded case of matter diffusing between two solid walls. This case is essentially different from the case of an unbounded atmosphere, but an equivalence may exist for short diffusion times. Saltzman (1962) has shown that for a finite upper bound to be insignificant  $t \ll Z_n^2/2D_n$  or  $\tau_n \ll 0.5$ . Dr. Atesman comments on effects of the wall detected for  $\tau_n > 0.15$ . Hence the range of dimensionless dispersion time used by the author is necessarily inadequate to criticize the results of the Lagrangian similarity hypothesis.

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#### AUTHOR'S CLOSURE

The author thanks Professors F. H. Chaudhry and R. N. Meroney for their thoughtful comments. The Batchelor's universal constant  $c$  is always less than unity as shown in Figs. 27 and 28 as a function of dispersion time. These values are calculated by using equations (15)–(17) along with the slopes of the first longitudinal moments. The value of  $c$  converges to Chatwin's constant value of 0.5615 for long dispersion times in a neutral atmosphere.

In a similar manner, the Batchelor's universal constant  $b$  is obtained by using equations (15) and (17) along with the slopes of the first vertical moments. The results are shown in Figs. 25 and 26.  $b = k$  with no restriction on height of release, but its value decreases with increasing stability.

The author agrees with the commentators that the initial condition  $\bar{X} = 0$  at  $t = 0$  for elevated sources is in error, but this error does not effect the values of Batchelor's universal

constants, since the slopes of the first moments are used instead of the integrated form of the equations to obtain them.

The present calculations are for small dispersion times. The present Eulerian diffusion model shows that the Lagrangian similarity theory is only applicable to long dispersion times, and the present calculations converge to the constant values that were obtained previously for long dispersion times.

Throughout the present calculations the upper boundary does not effect the results as shown in Figs. 4b and 17 because no dispersant reaches the neighborhood of the upper boundary for  $\tau_n \leq 0.15$ . Therefore, the present Eulerian diffusion model is actually predicting an unbounded atmosphere up to  $\tau_n = 0.15$ .

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